HYPERBOLIC EQUATION OF HEAT CONDUCTION FOR DISPERSED SYSTEMS

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The possible use of a hyperbolic equation to describe heat conduction in dispersed systems is analyzed.

In calculations relating to conductive heat transfer, a dispersed system has usually been considered as a uniform medium with certain effective characteristics λ, c, ρ. This type of approach has yielded excellent practical results when analyzing steady or slowly-varying states. However, in recent years the tendency toward the practical use of high-intensity heat transfer with dispersed coolants has greatly increased interest in the study of essentially transient processes. Experimental data relating to heat transfer between a solid surface and a dispersed coolant for small Fo numbers differ considerably from the values derived by using the classical heat-conduction equation [1, 2, etc.]. The theory developed in [3] and attempts at making an exact computer calculation of heat conduction through a layer of regularly-shaped particles [4, 5] gave no results adequate for practical use. Satisfactory agreement with experimental data was obtained in [6] and [7] by making additional assumptions as to the mechanism of the process ("contact" resistance in [6], the possibility of separating the effective thermal conductivity into two additive constituents corresponding to individual phases in [7]).

The possible use of a hyperbolic heat-conduction equation [9, 10] was discussed in [8]; this may be derived by using a law of heat propagation differing from the classical equation

$$\bar{q} = -\lambda\nabla\vartheta - \tau_r \frac{\partial\bar{q}}{\partial\tau} \tag{1}$$

and a heat-balance equation

$$-\nabla q = c\rho \frac{\partial\vartheta}{\partial\tau} \tag{2}$$

in the form

$$\frac{\partial\vartheta}{\partial\tau} + \tau_r \frac{\partial^2\vartheta}{\partial\tau^2} = \frac{\lambda}{c\rho}\Delta\vartheta. \tag{3}$$

The application of Eqs. (1)-(3) to dispersed media has an obvious physical meaning.* In a transient process, in fact, the temperatures of the individual phases at points quite close together are in general very different. Local heat transfer between the phases arising in this way retards the development of a temperature field in a heterogeneous medium. It is quite clear that τ_r (the relaxation time) represents the characteristic time required for equalizing the temperatures between the phases. An approach of this kind, which allows for the local relaxation of the temperatures in the dispersed system, clearly enables us to make a theoretical analysis of conductive heat-transfer processes for far shorter times and greater temperature gradients than would be possible using the classical heat-conduction equation.

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* The use of Eqs. (1)-(3) for uniform (homogeneous) media and their physical meaning and consequences were discussed in [9, 10, 11]. A hypothesis as to the finite velocity of heat and mass propagation in capillary-porous solids was proposed earlier in [12] and further considered in [13, 14], while a hyperbolic equation of transient mass transfer in dispersed systems was given in [15].

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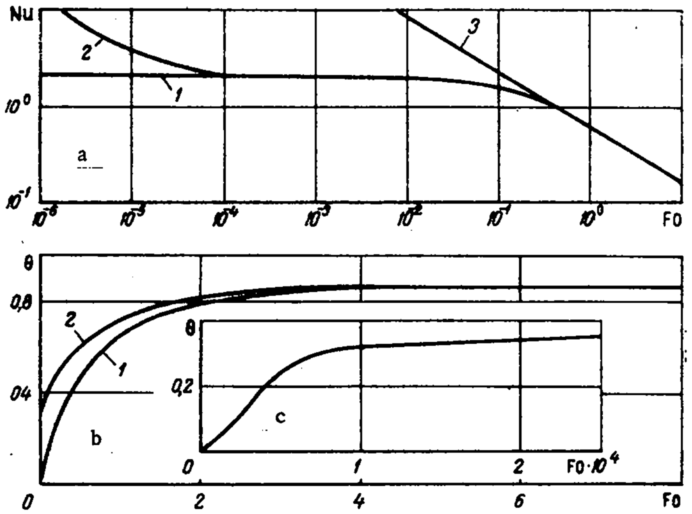

Fig. 1. Time dependence of the thermal fluxes (a) and temperatures (b, c): a) 1) solution of the hyperbolic equation; 2) solution of the system (15)–(16); 3) solution of the parabolic equation; b) 1) time dependence of θ_1; 2) time dependence of θ_2; c) time dependence of θ_2 for small Fo.

Let us apply Eqs. (1)-(2) in order to study a particular case: transient heat transfer between a surface and a compact layer of stationary, impenetrable material (i.e., one not allowing a stream of gas to pass completely through it), comprising a dispersed solid-particle—gas system, since experimental data have been published for precisely this case.

We consider the heat-conduction process in a semiinfinite layer of dispersed material. We supplement Eqs. (1) and (2) with the initial and boundary conditions:

$$q(x,\ \tau)=0 \quad \tau=0,\ x>0, \tag{4}$$

$$\vartheta(x,\ \tau)=0 \quad \tau=0,\ x>0, \tag{5}$$

$$\vartheta(x,\ \tau)=\vartheta_0 \quad x=0,\ \tau>0. \tag{6}$$

The solution to the problem (1), (2), (4)-(6) was derived in [11], and for a thermal flux takes the form

$$q=\vartheta_0\left(\frac{\lambda\rho c}{\tau_r}\right)^{1/2}\exp\left(-\frac{\tau}{2\tau_r}\right)I_0\left\{\frac{\tau}{2\tau_r}\left(1-\frac{x^2\tau_r}{\frac{\lambda}{c\rho}\tau^2}\right)^{\frac{1}{2}}\right\} \tag{7}$$

or in dimensionless variables

$$\mathrm{Nu}=\mathrm{Fo}_r^{-\frac{1}{2}}\exp\left(-\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right)I_0\left\{\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\left(1-\frac{y^2\mathrm{Fo}_r^2}{\mathrm{Fo}^2}\right)^{\frac{1}{2}}\right\}, \tag{8}$$

where

$$\mathrm{Fo}_r=\frac{\lambda\tau_r}{c\rho d^2}. \tag{9}$$

The time τ_r required for the temperature to become equalized between the gas and the particles may be determined from the equation of thermal balance between the phases, considering (to a first approximation) that the temperature drop inside the particles is negligible and that heat transfer between the particles and the gas obeys a linear law

$$c\rho\frac{\partial\vartheta}{\partial\tau}=-\alpha^*s\vartheta. \tag{10}$$

Hence

$$\vartheta=\mathrm{const}\exp\left(-\frac{\tau\alpha^*s}{c\rho}\right). \tag{11}$$

It is clear from Eq. (11) that the time constant of the process of interphase heat transfer

$$\tau_r = \frac{c\rho}{\alpha^* s}. \tag{12}$$

Substituting the values of τ_r, $\alpha^* = \mathrm{Nu}^*\lambda/d$, $s = 6(1-\varepsilon)/d$ into (9) we obtain

$$\mathrm{Fo}_r = \frac{1}{6(1-\varepsilon)\,\mathrm{Nu}^*}. \tag{13}$$

Putting y = 0 in (8)

$$\mathrm{Nu} = \mathrm{Fo}_r^{-\frac{1}{2}} \exp\left(-\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right) I_0\left(\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right). \tag{14}$$

We see from (14) that as $\mathrm{Fo} \to 0$ the thermal flux is limited: $\mathrm{Nu} = \mathrm{Fo}_r^{-1/2}$. As $\mathrm{Fo} \to \infty, \mathrm{Nu} \to (\pi \mathrm{Fo})^{-1/2}$, i.e., we obtain the ordinary solution of the classical heat-conduction equation for a semiinfinite solid (Fig. 1a).

It is of fundamental interest to compare the solution of the hyperbolic equation (14) with existing theoretical and experimental data regarding transient heat transfer between a surface and a compact layer of dispersed material. For this we make use of the results of [16], the authors of which analyzed the system of differential equations for transient heat conduction in a dispersed system of the dispersed solid-particles–gas type; in our own nomenclature this takes the form:

$$\mathrm{Fo}_r \frac{\partial \theta_1}{\partial \mathrm{Fo}} = \theta_2 - \theta_1, \tag{15}$$

$$\mu\,\mathrm{Fo}_r \frac{\partial \theta_2}{\partial \mathrm{Fo}} = \mathrm{Fo}_r \frac{\partial^2 \theta_2}{\partial y^2} - \theta_2 + \theta_1. \tag{16}$$

The solution to this system of equations for y = 0 and $\mathrm{Fo} > 0.007$ takes the form

$$\mathrm{Nu} = \left(\frac{\mu}{\pi \mathrm{Fo}}\right)^{\frac{1}{2}} \exp\left(-\frac{\mathrm{Fo}}{\mathrm{Fo}_r}\right) + \mathrm{Fo}_r^{-\frac{1}{2}} (1+\mu)^{\frac{1}{2}} \exp\left(-\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right) I_0\left(\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right) \tag{17}$$

and as indicated in [15] agrees satisfactorily with existing experimental data. It is not hard to see that as $\mu \to 0$ (in practice $\mu = 10^{-3}$) Eqs. (14) and (17) coincide. This simultaneously indicates that the solution to the hyperbolic equation agrees satisfactorily with the experimental data presented in [16]. It was shown in the reference cited that $\{6(1-\varepsilon)\mathrm{Nu}^*\}^{1/2} = 2$. Substituting this value in (13) we obtain $\mathrm{Fo}_r = 0.25$. Naturally for $\mathrm{Fo} \gg \mathrm{Fo}_r$ it is legitimate to use the ordinary classical Fourier equation, while for $\mathrm{Fo} < \mathrm{Fo}_r$ we must consider the hyperbolic equation.

We note that for small values of μ the term $\mu \mathrm{Fo}_r \partial\theta_2/\partial \mathrm{Fo}$ may be neglected even in the original system (15)–(16). This follows both from physical considerations (the term $\mu \mathrm{Fo}_r \partial\theta_2/\partial \mathrm{Fo}$ characterizes the intrinsic heating of the gas and is only appreciable for very short times, before the influence of the solid phase has made itself felt) and from calculations of the thermal fluxes and temperature fields over the individual phases, which we carried out by solving the system (15)–(16) numerically on a computer, using the grid (network) method. As a numerical example Fig. 1b, c illustrates the time dependence of the temperatures θ_1 and θ_2 for y = 0.5. We see from the figure that even for fairly small values of Fo the heating of the gas phase determined by the term $\mu \mathrm{Fo}_r \partial\theta_2/\partial \mathrm{Fo}$ comes to an end. The rise in temperature is then largely governed by the solid phase and the intensity of interphase heat transfer. Here the derivatives $\partial\theta_1/\partial \mathrm{Fo}$ and $\partial\theta_2/\partial \mathrm{Fo}$ are quantities of the same order and the term $\mu \mathrm{Fo}_r \partial\theta_2/\partial \mathrm{Fo}$ may be neglected for $\mu \ll 1$. The solution of the simplified system (15)–(16), in which the term $\mu \mathrm{Fo}_r \partial\theta_2/\partial \mathrm{Fo}$ is omitted, was derived by the method of integral transformations, and for the case of small y (important in our own discussions) takes the form

$$\mathrm{Nu} = \mathrm{Fo}_r^{-\frac{1}{2}} \exp\left(-\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right) I_0\left(\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right) - \frac{y}{\mathrm{Fo}_r} \exp\left(-\frac{\mathrm{Fo}}{\mathrm{Fo}_r}\right). \tag{18}$$

For y = 0 Eqs. (8) and (18) coincide.

Thus the use of the hyperbolic equation for describing the transient heat conduction of a dispersed medium has enabled us to derive a computing relationship Nu = f(Fo) in satisfactory agreement with experimental data. There is no longer any need to make special assumptions as to the mechanism of heat transfer in the layer and between the layer and the surface. Clearly it is also desirable to use the hyperbolic heat-conduction equation for other classes of dispersed systems; further theoretical and experimental investigations are required for this.

NOTATION

q	is the thermal flux;
$c\rho$	is the volumetric specific heat of the system;
x	is the coordinate;
$y = x/d$	is the dimensionless coordinate;
d	is the particle diameter;
$s = 6(1-\varepsilon)/d$	is the surface area of the particles in unit volume;
ϑ	is the temperature;
$\theta = \vartheta/\vartheta_0$	is the dimensionless temperature;
τ	is the time;
τ_r	is the relaxation time;
ε	is the porosity;
α, α^*	are the heat-transfer coefficient between the dispersed material and the surface and interphase heat-transfer coefficient respectively;
λ	is the effective thermal conductivity of the system;
$\mathrm{Nu} = \alpha d/\lambda$;	
$\mathrm{Nu}^* = \alpha^* d/\lambda$	are the Nusselt numbers;
$\mathrm{Fo} = \lambda\tau/c\rho d^2$	is the Fourier number;
Fo_r	is the dimensionless relaxation time;
$I_0(x)$	is the modified Bessel function of the first kind and the first order;
$\mu = c_2\rho_2/c_1\rho_1$	is the dimensionless parameter.

Subscripts

1	denotes solid phase;
2	denotes gas phase.

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